neglecting the gas phase energy equation, not the solid phase energy equation as done in Ref. 1. In addition, the attempt to account for solid phase heat transfer in Eq. (14) of Ref. 1 is inadequate because of the coupled nature of the equations.

Next we consider Liu's treatment of the equations. After transforming by a similarity variable there results Eq. (10) of Ref. 1. In this equation the time derivative has disappeared, thus losing part, but not all of the transient behavior. That the time derivative should be retained is evident from the fact that the Damköhler number is a function of time. In effect, Liu has made the assumption of local similarity[3] by neglecting the time derivative. The two-dimensional (η, t) problem was thus reduced to one in one independent variable (η) where the time (or Damköhler number) enters only as a parameter. It is well known[4,5] that gas-phase diffusion flames in one dimension exhibit multiple (or nonunique) solutions. It is reasonable to expect the same in heterogeneous diffusion flames, as shown in Ref. 1. It is also true that unique solutions occur,[5] but these were overlooked in Ref. 1.

Finally we consider the interpretation of the multiple (or nonunique) solutions. Liu[1] maintains that they correspond to a system stability, allowing the system to stabilize in either a weak or strong combustion mode. This is certainly true in the real one-dimensional case (characterized by a constant Damköhler number). But in the present case, where the Damköhler number is time dependent, we suggest that the system must pass to an unique equilibrium temperature corresponding to an infinite Damköhler number and a strong combustion mode. Physically, the nonunique solutions cannot exist in this case, but appear only as a consequence of the local similarity hypothesis. It is worthwhile to note that this hypothesis breaks down for nonunique solutions because such solutions acquire infinite time-rate of changes. This is contrary to the original hypothesis of slow changes on the time scale inherent in the local similarity hypothesis. Our interpretation of the nonunique solutions is related to the transient behavior of the system, and particularly to that part of the transient behavior which is lost by the local similitude. The nonunique solution corresponds to a very abrupt transition (or ignition) from the kinetic controlled regime (small Damköhler number) to the diffusion controlled regime (large Damköhler number). This is in contrast to the unique type of solution where the transition is a slow one. The author has recently obtained closed form analytic solutions to the problem under discussion.[6] These solutions bear out the above expressed ideas when compared with exact numerical solutions[7,8] for the complete time dependent problem.

One further point is to be made. The problem of fire prevention in space capsules is both physically and mathematically similar to the problems of solid fuel and solid propellant ignition. There already exists a large body of literature (Refs. 2 and 6–10, to cite a few) covering both heterogeneous and gas phase reactions. The problem of Ref. 1 has been specifically treated by Williams.[7] He considered the case of a solid fuel suddenly exposed to an oxidizing atmosphere where both media are initially at the same temperature. Recently, Waldman and Summerfield[8] have extended Williams' theory to cases where the two media are initially at different temperatures. Both studies considered the full set of equations.

In conclusion, an examination of Ref. 1 has revealed serious flaws in formulation, analysis, and conclusions, Moreover, useful solutions to the problem posed in Ref. 1 can be obtained from the existing literature.

References

[1] Liu, S. W., "Onset of Surface Combustion in Still Atmosphere," *AIAA Journal*, Vol. 7, No. 4, April 1969, pp. 734–736.

[2] Price, E. W. et al., "Theory of Ignition of Solid Propellants," *AIAA Journal*, Vol. 4, No. 7, July 1966, pp. 1153–1181.

[3] Hayes, W. D. and Probstein, R. N., *Hypersonic Flow Theory*, Academic Press, New York, 1959.

[4] Fendell, F. E., "Ignition and Extinction of Initially Unmixed Reactants," *Journal of Fluid Mechanics*, Vol. 21, 1965, pp. 281–303.

[5] Chung, P. M., Fendell, F. E., and Holt, J. F., "Nonequilibrium Anomalies in the Development of Diffusion Flames," *AIAA Journal*, Vol. 4, No. 6, June 1966, pp. 1020–1026.

[6] Waldman, C. H., "Theory of Heterogeneous Ignition," unpublished notes; abstract submitted for presentation at the AIAA 8th Aerospace Sciences Meeting, New York, January 19–21, 1970.

[7] Williams, F. A., "Theory of Propellant Ignition by Heterogeneous Reaction," *AIAA Journal*, Vol. 4, No. 8, Aug. 1966, pp. 1354–1357.

[8] Waldman, C. H. and Summerfield, M., "Theory of Propellant Ignition by Heterogeneous Reaction," *AIAA Journal*, Vol. 7, No. 7, July 1969, pp. 1359–1369.

[9] Hermance, C. E., Shinnar, R., and Summerfield, M., "Ignition of an Evaporating Fuel in a Hot Oxidizing Gas, Including the Effect of Heat Feedback," *Astronautica Acta*, Vol. 12, 1966, pp. 95–112.

[10] Waldman, C. H., "Theoretical Studies of Diffusion Flame Structures," Ph.D. thesis, Dec. 1968, Princeton Univ., Princeton, N.J.

Reply by Author to C. H. Waldman

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In the preceding Comment, a number of questions were raised concerning Ref. 1 of that Comment. In answering, it is first to be pointed out that there can be a difference between the model assumed for solid fuel combustion studies intended for propulsion and that of capsule fire. In Ref. 1, we considered the case where internal heating, due to electrical equipment, raised the temperature of combustible material to a level where appreciable surface reactions occurred. It was assumed that the temperature of a solid was raised instantaneously and uniformly to a given level before the reaction proceeded and remained at that level during combustion. Thus, there is no net local heat transfer in the solid. It was therefore appropriate in our model to direct attention mainly to the gas phase flowfield using the constant surface temperature as a parameter. When such pointwise solutions were used in the discussion of the system stability, they served only to indicate qualitative trends based on the over-all heat and mass balance. This is in contrast to the models assumed in all the references cited by Waldman of solid fuel surface combustion studies orientated to rocket applications in which the only energy source was that due to combustion at the surface and heat was lost primarily by conduction into the relatively cool solid.

The question was also raised by Waldman of the use of the similarity transformation which tacitly invoked the local similarity assumption. It is agreed that something may have been lost, but the discrepancy will not be large except when local true time derivatives of the dependent variables depart appreciably from that depicted by similarity. The materials of concern in a space capsule are by choice all of low flammability. They are therefore a vastly different kind of "fuel" than those considered in the solid fuel combustion for rocket propulsion. Drastic changes of mode in the surface reaction

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of these materials are very unlikely. Since the reaction takes place only at the surface, the Damköhler number appears only in the surface boundary condition. In the flowfield the Damköhler number is zero. When the Arrhenius rates, given for such materials are used, the Damköhler number is strongly dependent on surface temperature and only weakly on time. It is well known that the similarity solution of the type used in Ref. 1 is not valid for $t = 0$. When moderate values of t are considered, the Damköhler number is well behaved (monotonically). So, even in the transitional regime where the essential parametric variation is in the surface temperature, the results based on this quasi-steady boundary condition remained meaningful if they are interpreted consistent with the surface boundary condition of Eq. (13) of Ref. 1. In the diffusion-limited regime this boundary condition is well defined regardless of surface temperature or time, and so are the corresponding results. It is true that these solutions do not provide the exact time history of the reaction path. Such results were not intended in the studies of Ref. 1. The result for equilibrium surface temperature, however, is well defined by that study.

It is further pointed out that our consideration of system stability is based on the surface heat and mass balance and is not the same as that of the multiple solutions in nonequilibrium gaseous combustion analyses (Refs. 4 and 5 of the preceding Note). In the latter analyses, the multiple solutions are a result of local excursion of the Damköhler numbers in the flowfield. A more valid comparison with our analysis is to Spalding's work noted in Ref. 1, in which system stability had been considered in a similar manner for the surface combustion of carbon in a convective flow. References 6 and 8 of the preceding Note are not yet in the open literature; it is therefore not possible at this writing to include their merits in the discussion.

Then, for the purposes intended, the analyses and results of Ref. 1 are valid. As in many examples of boundary layer and related studies, solutions using local similarity and using the complete partial differential system each has its own merits, depending on the nature of problem in question. Different approaches often aid the over-all understanding. It is in part under such a premise that the results of Ref. 1 were offered.

Comment on "Chemistry of Electrons in Pure-Air Hypersonic Wakes"

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Introduction

Measurements of the electron line densities appearing in the wakes of hypersonic pellets[1] have provided stimulus for concomitant theoretical computations which are designed to test suitable chemical-gas-dynamical wake models. In the model adopted recently by Sutton,[2] related to earlier work of Lin and Hayes,[4] a chemically reacting laminar flowfield obtained by the streamtube method for a sphere with suitably streamlined afterbody (to avoid the difficulties associated with the details of base flow) is used to calculate input specie flux densities to a wake that assumes the one-third-power-with-distance radius growth law[5] and an infinite viscous dissipation rate. We refer the reader to the articles of Lin and Hayes,[4] Sutton,[2] and others,[7–10] for full discussion of this homogeneous mixing model and of other pertinent wake phenomena.

The purpose of the present comment is to draw attention to a possibly significant difference between the homogeneous mixing model employed by Sutton[2] and the original homogeneous mixing model examined by Lin and Hayes,[4] and to the presence of possible internal inconsistencies in the computational results so obtained. In addition, we question the general legitimacy of Sutton's application of the homogeneous mixing model to problems in wake chemistry by drawing attention to a number of features of the model which are apparently incompatible with observation.

Homogeneous Mixing

Restricting our attention to the far wake, subsequent to breakthrough,[6] we have from Lin and Hayes,[4]

$$\left.\frac{dn_i}{dt}\right|_{\text{mixing}} = \frac{2}{3}\,[n_i(\infty) - n_i(t)]t^{-1} \tag{1}$$

for the time rate of change due to ingestion and wake expansion of the number density, $n_i(t)$, of the ith wake specie, where $n_i(\infty)$ is the ith specie ambient number density. On the other hand, we have from Sutton [Ref. 3, Eq. (21), correcting a typographical error], for the same mixing derivative,

$$\left.\frac{dn_i}{dt}\right|_{\text{mixing}} = n(t)\,\frac{n_i(\infty)}{n(\infty)}\left(\frac{2}{3}\,t^{-1} - \frac{1}{T}\,\frac{dT}{dt}\right) - \frac{2}{3}\,n_i(t)t^{-1} \tag{2}$$

where $n(t)$ and $n(\infty)$ are the wake and ambient total number densities, respectively. Here, Eq. (1) follows from consideration of the derivative of $n_i(t) = N_i(t)/V(t)$, with dN_i/dt (ingestion) $= n_i(\infty)dV/dt$ where the change in wake reference volume is determined by the one-third-power-with-distance wake radius growth law.[5] Equation (2) follows from similar consideration with, however, dN_i/dt (ingestion) $= [n_i(\infty)/n(\infty)]dN/dt$, where the rate of change of the total number of wake particles, dN/dt, in the reference volume $V(t)$ is determined in terms of temperature and volume changes by the ideal gas law. The latter procedure is an approximation and evidently assumes that the rate of change of wake particles due to chemical reaction is negligibly smaller than the particle ingestion rate due to wake expansion.

We now examine the conditions under which the mixing rates of Eqs. (1) and (2) can differ. Such difference will evidently depend on the manner in which the temperature history, $T(t)$, in Eq. (2) is obtained. Following Sutton, this temperature history can be determined by equating the rate of change of wake enthalpy per unit length to the ingested enthalpy plus that produced by chemical reaction, i.e.,

$$\left(\frac{C_p}{N_0}\right)N\,\frac{dT}{dt} + \left(\frac{C_p}{N_0}\right)T\,\frac{dN}{dt} = \left(\frac{C_p}{N_0}\right)T_\infty\,\frac{dN'}{dt} + \sum_i D_i\left.\frac{dN_i}{dt}\right|_{\text{chemistry}} \tag{3}$$

Here, N_0 is Avogadro's number, C_p the molar specific heat at constant pressure, dN'/dt the rate of ingestion of ambient particles, T_∞ the ambient temperature, and D_i the ith specie dissociation energy. Assuming with Sutton that $dN/dt = dN'/dt$ (i.e., neglecting particle number change due to chemical reaction) and introducing the ideal gas law, from which we obtain

$$kT(dN/dt) = P(dV/dt) - Nk(dT/dt) \tag{4}$$

we see that Eq. (3) takes the form

$$\frac{1}{T}\,\frac{dT}{dt} = \frac{2}{3}\left(1 - \frac{T(t)}{T_\infty}\right)t^{-1} + \frac{1}{T_\infty}\sum_i\left(\frac{D_iN_0}{C_p}\right)\frac{1}{N(t)}\left.\frac{dN_i}{dt}\right|_{\text{chemistry}} \tag{5}$$

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